Explore To Find the Optimal Portfolio in The Financial Market

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Abstract: With the existence of liquidity and volatility in stock and security markets, it is vital to establish efficient investment portfolios to control risks and yield profits. Thus, this project mainly employs the Markowitz model and Index model to implement portfolio analysis with respect to historical daily total return data for ten stocks from 2001 to 2021 and one equity index S&P500, to work out minimal risk frontier, minimal risk portfolio, maximal sharp ratio portfolio, efficient frontier, capital allocation line, and minimal return frontier of each model. Simultaneously to further analysis, the aforementioned problems using the Markowitz model and Index model are implemented and solved with five additional optimization constraints. Through empirical results with diverse constraints, we find out that the Markowitz model's returns are always higher than the Index's ones under five constraints in the minimum variance graph. Meanwhile, the index model can always achieve a higher sharp ratio than the Markowitz Model in the maximum sharp ratio. Thus, this project summarizes the regularity and differences of various kinds of diversification in both two models.

1. Introduction

Portfolio management is significant to investors because investing a large amount of money in one stock is highly risky, but proper choices of stocks in a portfolio lead to profits. It is easy to choose some stocks, futures, and treasuries. However, the hard part is evaluating whether this portfolio is worth investing in. Risk and return are tradeoffs that higher returns require higher risk. In the meanwhile, the chances are that investors lose money. Markowitz Model and Index model are two different models to evaluate portfolios. Markowitz Model assumes that people are risk-averse, which means investors prefer a less risky portfolio than a riskier portfolio with the same expected return. The Index Model connects the capital asset pricing model, which gives a prediction between expected return and risk of an asset and realizes return. By putting the different amounts of money into each stock, the result of return and standard deviation will be different. What return can a portfolio achieve with a minimal standard deviation? What is the highest sharp ratio by using different models?

To determine the difference between the Markowitz model and Index model, the portfolio chosen includes four different industries Consumer Cyclical- Amazon, Technology-Apple, Citrix System, Financial Services- JPMorgan Chase, Berkshire Hathaway, The progressive Operation, Industrial Services- United Parcel Service, FedEx Corporation, J.B Hunt Transport Services, and Landstar System. Industrials and financial services companies take a large portion of the portfolio. Therefore, these two sectors play a large role in overall return and standard deviation.

Markowitz's Mean-Variance Model is a useful instrument for choosing the portfolio [1]. However, it requires quite a lot of data and calculations. Index Model is therefore put forth to resolve these problems [2]. It enormously simplifies the calculations as well as the processes of data cultivation.

Some researchers tried to apply the index model to real life. Index model was used to analyze the relationship between Shanghai and Shenzhen 300 index and Ping`an Bank [3]. The similarity between the results of the index model and the test of the market justified the excellent applicability of the index model in the Chinese stock market [4]. The return of the pension fund was calculated by the index model [5]. Although several setbacks in the index model were pointed out [6], empirical research still demonstrated that the index model could perfectly explain some industry risks and excess returns. With the help of the Index Model, the linear relationship between Shanghai and Shenzhen 300 index and Finance, IT, and real estate industry was found [7]. Stocks from 2013 to 2017 were analyzed [8]. Index Model was carefully applied to the real estate industry [9]. Indian stock market also well corresponded to Index Model [10]. They all well clearly showed the successes and short backs of the Index Model, but few of them used international stocks. More importantly, they merely used data in several years, which lacked some cogency if the applicability of the Index Model was questioned in the long run. In this paper analyzes data of several international companies from 2001 to 2021 and compares the results of the Markowitz Model and the Index Model.

The purpose of this paper is to find the allowed portfolio region under five additional constraints with optimization inputs from the full Markowitz model ("MM") as well as from the exponential model ("IM"). Inferences and comparisons are made between the constraint sets for each optimization problem and between the general MM and IM models. The optimal inputs under both models are found by aggregating historical daily return data for 10 stocks over the last 20 years to monthly observations, combining a stock index (S&P 500) and a risk-free proxy for interest rates (1-month federal funds rate), and finding the efficient frontier, minimum risky portfolio, optimal portfolio, and minimum return under the five additional constraints based on the optimal inputs and monthly observations investment frontier. The final results are presented in the form of tables and graphs.

This paper will be presented in five sections. In one section, a short introduction to the research project of this paper will be presented. The data selected and the companies will be described in detail in the second paragraph. In the third part, the specific research methodology is presented. The fourth section presents the final results and the analysis. The fifth section will give the conclusion of the paper.

2. Data

The article collects a recent 20 years from 2001 to 2021 of historical daily total return data for ten stocks, including one equity index S&P500, and a proxy for the risk-free rate. S&P500, refers to a record of the average U.S. stock market since 1957, contenting 500 publicly traded companies in the United States. The fundamental information of each company and stock price graph for 20 years are presented as follows.

Amazon.com inc. is a multinational e-commerce company in the United States, founded in 1995. Amazon is one of the earliest companies operating e-commerce on the Internet. It only engaged in online book sales initially, and now it has expanded to a wide range of other products, including music and games, digital downloads, electronics and computers, home gardening supplies, etc. It has become the second-largest Internet enterprise in the world. It also includes AlexaInternet, Lab126, and Internet Movie Database (IMDB). Amazon ranked 4th in the Forbes 2020 Top 100 Global Brand Value list released in July 2020. The price graph from the launch date to 2021 is expressed as follows (see Figure 1).

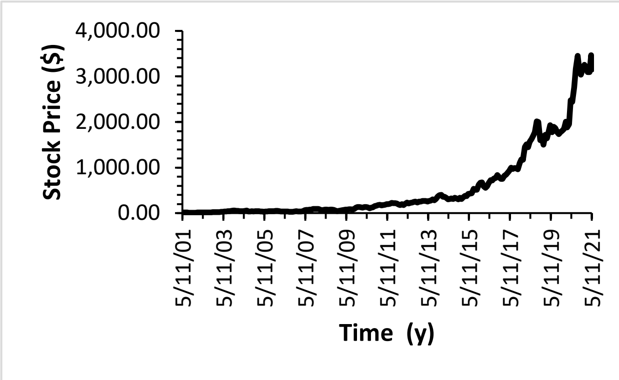

Figure 1. The stock price of AMZN from 2001 to 2021.

Apple is an American high-tech company that is famous for innovation founded in 1976 by Steve Jobs, Steve Wozniak, and Ron Wayne. Its core business is electronic technology products, including well-known products like Apple II, Macintosh computer, Macbook notebook computer, iPod music player, iTunes Store, iMac all-in-one, iPhone and iPad tablet computer, and so on. Apple ranked 6th in the Fortune global 500 lists in 2021. The price graph from the launch date to 2021 is expressed as follows (see Figure 2).

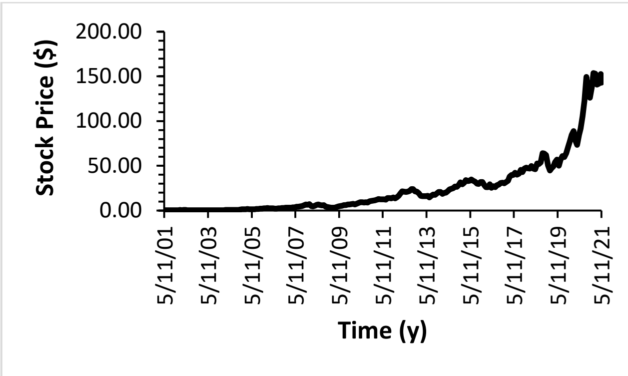

Figure 2. The stock price of AAPL from 2001 to 2021.

Citrix is a high-tech company dedicated to cloud computing virtualization, virtual desktop, and remote access technology. Citrix came up with the BYOD (Bring Your Own Device) prevailing now. Citrix's vision, established in 1997, was to make the information as easy and convenient as a phone call, accessible to anyone, anytime, anywhere. Citrix is the leading provider worldwide of application services, software solutions, and digital independence. The price graph from the launch date to 2021 is expressed as follows (see Figure 3).

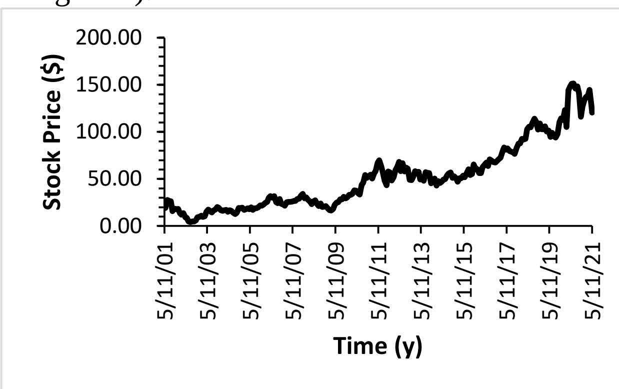

Figure 3. The stock price of CTXS from 2001 to 2021.

JPMorgan Chase was formed in 2000 by the merger of Chase Manhattan Bank and J.P. Morgan & Co. JPMorgan Chase is a multinational financial service institution and one of the largest banks in the United States, headquartered in New York, the United States, with total assets of 2.5 trillion dollars and total deposits of 1.5 trillion dollars, accounting for 25% of the total deposits in the United States. It has more than 6,000 branches and businesses in more than 60 countries, including investment banking, financial transaction processing, investment management, commercial, financial services, personal banking, and so on. The price graph from the launch date to 2021 is expressed as follows (see Figure 4).

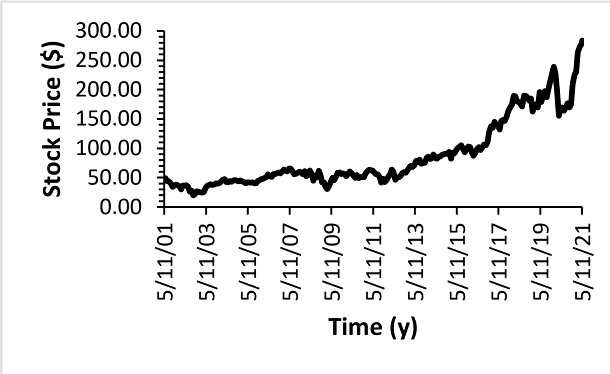

Figure 4. The stock price of JPM from 2001 to 2021.

Berkshire Hathaway, founded by Warren Buffett in 1956, is an insurance company with many other businesses, among which property and disaster insurance based on direct premiums and reinsurance amounts are the most important. Berkshire Hathaway has many subsidiaries, including GEICO, the sixth-largest auto insurance company in the United States; General Re, one of the four largest reinsurance companies in the world. The price graph from the launch date to 2021 is expressed as follows (see Figure 5).

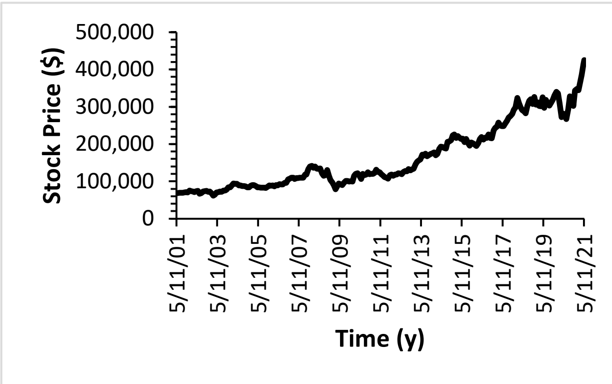

Figure 5. The stock price of BRK/A from 2001 to 2021.

Progressive Insurance was founded in 1937 by Jack Green and Joe Lewis. The business is mainly divided into three parts: personal vehicle insurance through insurance agent distributors and direct selling company two channels to provide private cars, motorcycles, ships, and other transportation insurance; commercial vehicle insurance through insurance agent distributors to the enterprise owner's truck and other commercial vehicle liability insurance including liability insurance for clients such as small regional banks and corporate executives. The price graph from the launch date to 2021 is expressed as follows (see Figure 6).

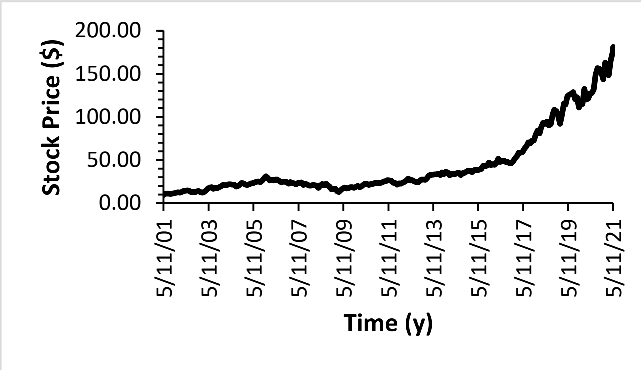

Figure 6. The stock price of PCG from 2001 to 2021.

UPS, United Parcel Service, Inc. founded in 1907 and headquartered in Atlanta, Georgia, is a global leader in the package and includes cargo transportation, international trade facilitation, advanced technology deployment, and other solutions designed to improve the efficiency of global business management. UPS employs 495,000 people in more than 220 countries and territories. UPS turnover reached $74 billion in 2019 and ranked 48th in the Forbes global Brand Value list 2020. The price graph from the launch date to 2021 is expressed as follows (see Figure 7).

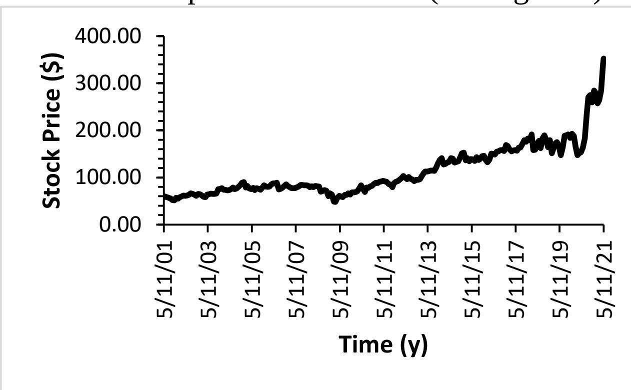

Figure 7. The stock price of UPS from 2001 to 2021.

FedEx is an international express delivery group, providing overnight express, ground express, heavy cargo delivery, document copying, and logistics services, headquartered in Memphis, Tennessee. FedEx is a subsidiary of the Federal Express Corporation with more than 26,000 employees. The price graph from the launch date to 2021 is expressed as follows (see Figure 8).

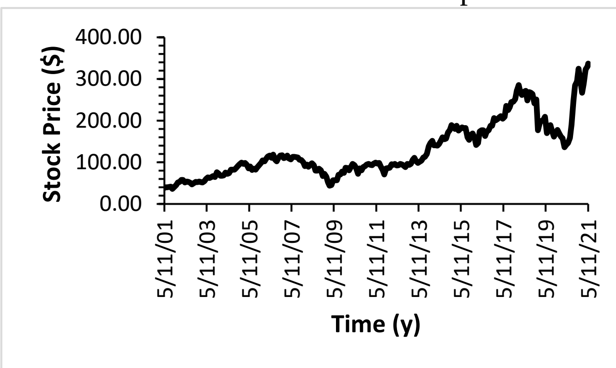

Figure 8. The stock price of FDX from 2001 to 2021.

J.B. Hunt Transportation Services is a trucking company founded by Johnny Brian Hunt. Nowadays, with revenues of $2 billion, J.B. Hunt is one of the biggest trucking companies. The main businesses are large two-wheel trailers, and the company provides transportation services across the United States, Mexico, and Canada. The company employs more than 16,000 people and has more than 11,000 trucks in operation. In addition, there are more than 47,000 trailers and containers. The price graph from the launch date to 2021 is expressed as follows (see Figure 9).

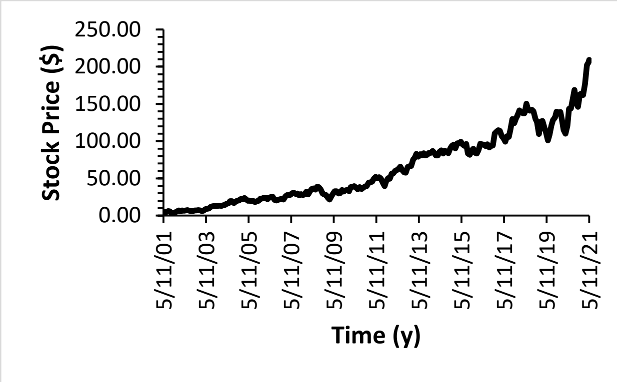

Figure 9. The stock price of JBHT from 2001 to 2021.

Landstar is a third-party logistics provider operating an asset-light platform that does not own any trucks but connects truck drivers with cargo shippers and independent sales agents. With very limited capital investment, Landstar generates a very high return on investment, around 30% to 40%, as well as very high cash flow. The price graph from the launch date to 2021 is expressed as follows (see Figure 10).

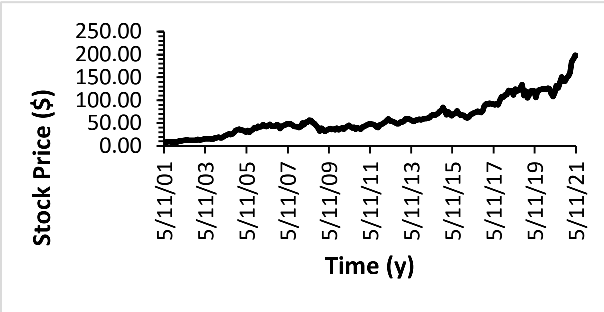

Figure 10. The stock price of LSTR from 2001 to 2021.

Excel implements the data processing with ten stocks and SPX index, during which the initial step is to convert daily data to monthly data, followed by return calculation, based on the last monthly statistic, and then minus one to get excess returns. The sum of total excess returns is deposed by the average function, multiplied by 12 to get annualized average return yearly. The beta and alpha are the slope and intercept of the relationship between each stock and SPX index, respectively.

Then, according to the residual return equation expressed as follows, the residual return is calculated based on previous excess return, beta, and alpha data.

$$Rr = Er - \beta \times Er - \alpha \tag{1}$$

In this equation, Rr, er refer to return rate and excess rate, respectively, and β, α refer to beta and alpha data mentioned before.

In addition, the correlation between stocks and SPX index are counted by correlation tool at data analysis function, shown as table 1

Table 1. Correlation among 10 stocks and SPX

	SPX	AMZN	AAPL	CTXS	JPM	BRK/A	PGR	UPS	FDX	JBHT	LSTR
SPX	1.000	0.485	0.542	0.437	0.697	0.523	0.502	0.575	0.614	0.521	0.495
AMZN	0.485	1.000	0.377	0.217	0.252	0.118	0.200	0.296	0.280	0.308	0.256
AAPL	0.542	0.377	1.000	0.332	0.244	0.173	0.240	0.231	0.330	0.268	0.287
CTXS	0.437	0.217	0.332	1.000	0.324	0.181	0.271	0.264	0.331	0.290	0.252
JPM	0.697	0.252	0.244	0.324	1.000	0.452	0.393	0.361	0.440	0.442	0.375
BRK/A	0.523	0.118	0.173	0.181	0.452	1.000	0.264	0.404	0.385	0.239	0.234
PGR	0.502	0.200	0.240	0.271	0.393	0.264	1.000	0.392	0.365	0.280	0.289
UPS	0.575	0.296	0.231	0.264	0.361	0.404	0.392	1.000	0.675	0.459	0.441
FDX	0.614	0.280	0.330	0.331	0.440	0.385	0.365	0.675	1.000	0.537	0.482
JBHT	0.521	0.308	0.268	0.290	0.442	0.239	0.280	0.459	0.537	1.000	0.590
LSTR	0.495	0.256	0.287	0.252	0.375	0.234	0.289	0.441	0.482	0.590	1.000

3. Method

We will use Index Model and Markowitz Model to study different portfolios with different constraints in order to find the best portfolio as well as compare the results. These results will be beneficial to investors to make wiser choices.

3.1 Index Model

Index Model contains several assumptions and one function.

The model assumes that there is only one macroeconomic factor that can contribute to the risk of the stock return, that almost all the stocks have positive covariance owing to their similar reaction to public events, that difference of such reactions exists which is controlled by β_i in the function and that the covariance of stocks is caused by different sensitivity, making covariance of single stock equals β multiply market variance.

The return of every stock can be explained in three components: market, firm-specific expected return, and firm-specific unexpected return.

The function of the Index Model is expressed as follow:

$$R_i = \alpha_i + \beta_i R_M + e_i \tag{2}$$

Where R_i refers to the excess return of the stock, namely $R_i = r_i - r_f$, R_M is the excess return of the market, namely $R_M = r_M - r_f$, α_i is the alpha of the stock or abnormal return, β_i is the stock's beta or sensitiveness of the stock to the market return, and e_i is the residual return.

3.2 Markowitz Model

Markowitz Model is a theory comprised of a series of hypotheses and functions.

It hypothesizes that investors are rational who only make decisions according to mean and variance, that investors are risk-averse, that investors pursue utility, that investors have the same expectations, and that investors will hold their investments for a certain time after their investment. In addition, in terms of market, Markowitz Model supposes that the capital market is efficient. That means the information of returns and risks is complete, and the prices of stocks reflect all the information investors can get. It also supposes that the supply is elastic, enabling investors to buy any number of stocks they want.

The total return of the portfolio is defined as follows:

$$\mathrm{E}(r_p) = \sum_{i=1}^{n} w_i E(R_i) \tag{3}$$

Where w_i refers to weights of stocks, $\mathrm{E}(R_i)$ refers to returns of different stocks and E(RP) refers to returns of the whole portfolio.

The total risks of the portfolio are defined as the variance of the portfolio:

$$\mathrm{Var}(r_p) = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j V_i V_j Cov(V_i, V_j) \tag{4}$$

Where w_i refers to weights of different stocks, $Cov(V_i, V_j)$ refers to the covariance of different stocks, and the $\mathrm{Var}(r_p)$ is the total variance of the portfolio.

The feasible set of the Markowitz Model is the set containing different portfolios with different weights. Different constraints will render different feasible sets. But, in terms of its efficient set, frontiers will have remained, and the other portfolios will be discarded. Such frontier contains efficient frontiers and inefficient frontiers. The contact point of these two frontiers is called the global minimum variance point, which is the portfolio with minimum risks given a certain constraint.

The CAL line is representative of the budget of investors. Here is its mathematical expression:

$$\mathrm{CAL} = r_f + \frac{E(R_m) - r_f}{\delta_m} \delta_p \tag{5}$$

where r_f is the return of the risk-free assets, δ_m refers to systematic risks, $E(R_m) - r_f$ refers to the excess return.

The tangent line of the efficient frontier is the CAL line chosen eventually. The points on the CAL line and efficient frontier above CAL line are portfolios that investors will finally choose according to their utility function.

3.3 Introduction to five constraints

Five different constraints will be used to simulate different circumstances so as to get different returns. w_i is referred to the weights of different stocks in the portfolio.

Constraint 1: This constraint is set to simulate Regulation T, which is a package of provisions. If investors want to borrow money to invest, they have to cater to several requirements. Investors must open a margin account if they want to buy securities with credit from broker-dealers. They are also required to keep 50% cash in their accounts. This is designed to ensure investors can repay their loans. In other words, the largest amount of money investors can borrow is half of the price they pay. Its function is expressed below:

$$\sum_{i=1}^{n} |w_i| \leq 2 \tag{6}$$

Constraint 2: This constraint simulates some constraints on weights which are usually set by the client. Its functional expression is shown below:

$$|w_i| \leq 1, for\ \forall i \tag{7}$$

Constraint 3: This optimization constraint is special, owing to its no constraint, which is designed to resemble the ideal investment environment.

$$\text{No constraints} \tag{8}$$

Constraint 4: This constraint is designed to simulate the representative restrictions of U.S. mutual funds: as long as the fund is not the participant of underwriting, it is prohibited from affecting short-term sales of securities.

$$w_i \geq 0, for\ \forall i \tag{9}$$

Constraint 5: To test the effects of some broad indexes, namely positive or negative effects, the weight of the index is controlled as zero. The result of the no-index portfolio will be compared with other results of portfolios with a certain percent of index:

$$w_1 = 0 \tag{10}$$

Where w_1 is the weight of the SPX

4. Result Analysis

We collect daily historical total return data from Yahoo Finance for the following 10 stocks: AMZN, AAPL, CTXS, JPM, BRK/A, PGR, UPS, FDX, JBHT, LSTR from 2001/11/05 to 2021/12/5 for 20 years and aggregate the daily data into monthly observations based on the s&p 500 index and a proxy for the risk-free rate (monthly federal funds rate). Finally, based on these monthly observations, the Markowitz and index models are used to calculate the efficient frontier, the minimum portfolio at risk, the optimal portfolio, and the minimum return frontier under five additional constraints and analyze the allowed portfolio area.

4.1 Markowitz Model Result

According to the following assumptions: (1) the securities market is considered efficient, and the prices of assets reflect their intrinsic values. (2) stock prices reflect all market information, and investors know the expected returns and standard deviations of various assets; (3) all investors are rational and seek to minimize the variance of asset portfolios; and (4) there is a correlation between the returns of various assets, which can be expressed in terms of correlation coefficients or covariances. The Markowitz model establishes an efficient frontier using expected returns, calculation methods for risk measures, and the theory of mean-variance. The Markowitz model establishes a mean-variance model for selecting the optimal asset portfolio using expected returns, risk measures, and the effective frontier theory.

The minimal variance frontier. The minimum variance can be achieved for a given level of expected return. By setting wi (wi=1, 2, 3.... .10, i.e., 10 stocks) as the variable unit of variation and set the standard deviation of the portfolio as the target for the planning solution, resulting in the minimum variance (table 1), and finally return should be set by creating dummy variables and changing the dummy variables continuously so that we can use solver table to find the minimal variance frontier. (Figure 1)

Table 2. Minimum variance of Markowitz model

Minimal variance			
	Return	StDev	Sharpe ratio
Constraint 1	7.15%	12.24%	0.584
Constraint 2	7.15%	12.24%	0.584
Constraint 3	7.15%	12.24%	0.584
Constraint 4	10.04%	13.09%	0.767
Constraint 5	13.20%	13.39%	0.986

The minimum variance frontier gives a graph of the minimum variance that a portfolio's expected return can achieve when the variance is smallest, and the risk is low. All individual assets lie to the right of the minimum variance frontier, i.e., a portfolio consisting of individual assets is inefficient. Based on figure 1, it can be seen that constraint 3 has the largest range of individual assets and constraint 4 has the smallest. The global minimum variance frontier components are called the efficient frontier of risky assets, and they provide the best risk-return portfolio.

Optimal risky portfolio. A well-diversified portfolio consists of individually risky assets, resulting in the lowest possible variance for the rate of expected return. We calculate the capital allocation line with the highest volatility (Sharpe) (table 2) by We calculate the capital allocation line with the highest volatility (Sharpe) from the previously calculated return and standard variance, that is, the steepest slope. The optimal risky portfolio p corresponds to the tangent CAL of the efficient frontier, and this CAL outperforms all other feasible routes.

Table 3. Maximum Sharpe ratio of Markowitz model

Maximum Sharpe ratio			
	Return	StDev	Sharpe ratio
Constraint 1	26.43%	18.70%	1.414
Constraint 2	33.18%	22.11%	1.501
Constraint 3	49.61%	32.25	1.539
Constraint 4	22.09%	17.62%	1.254
Constraint 5	23.89%	18.02%	1.326

A higher Sharpe ratio indicates a higher reward per unit of risk and the more desirable the investment vehicle is. A low Sharpe ratio implies that the fund is earning returns by taking higher risks. A high Sharpe ratio implies that the fund has a higher ability to diversify and reduce non-systematic risks and that there is room to increase returns. When the Sharpe ratio is above the CML, it indicates that the fund is outperforming the overall market performance. The Sharpe ratio is the highest (1.539) under the condition of constraint 3 (free problem), which means that without constraint, the risk per unit is higher, but it is also the most desirable investment instrument; while the Sharpe ratio of constraint 4 is the lowest, but it is also greater than 1, which means that it also has investment value, but the return it obtains is not as high compared to the other The Sharpe ratio of constraint 4 is the lowest, but it is also greater than 1.

4.2 Index Model Results

The model simplifies the problem of estimating the covariance matrix and enhances the analysis of the risk premium for the expected return on the security.

It explicitly decomposes risk into systematic risk and firm-specific components. Thus, it shows the power and limitations of diversification. In addition, it allows measuring these risk components for specific securities and portfolios.

The minimal variance frontier. Similarly, we can derive the minimum variance and find the minimum variance boundary by setting the changeable unit for the regression solution and determining the effective boundary according to the point of minimum variance. According to the figure, we can learn that the variance of constraint 1 and constraint 2 are more discrete than the other three constraints, indicating that the risk they need to take is larger than the other. In addition, from the minimum variance plots of the two models, it is easy to see that the Markowitz model calculates more data than the index model, which greatly reduces the number of measurables, so it is the preferred model in practical portfolio management.

Table 4. Minimal Variance of Index model

Minimal variance			
	Return	StDev	Sharpe ratio
Constraint 1	6.46%	12.41%	0.521
Constraint 2	6.46%	12.41%	0.521
Constraint 3	6.46%	12.43%	0.520
Constraint 4	10.24%	12.29%	0.788
Constraint 5	11.27%	13.21%	0.869

Optimal risky portfolio. First, establish the logic that change wi to find out the portfolio with the lowest variance to determine the Sharpe ratio, and then find the portfolio by solver table operation.

Table 5. Maximal Sharpe ratio of Index model

Maximal Sharp ratio			
	Return	StDev	Sharpe ratio
Constraint 1	28.58%	19.95%	1.433
Constraint 2	34.07%	22.35%	1.524
Constraint 3	60.91%	38.16%	1.596
Constraint 4	24.45%	19.16%	1.277
Constraint 5	26.95%	20.24%	1.331

As in the Markowitz model, the sharp ratio of constraints 3 (free problem) is the largest (1.596), and the sharp ratio of constraint 5 is the smallest, but both are greater than one, and all Sharpe ratios in the index model are greater than those in the Markowitz the Sharpe ratio in the index model is greater than that in the Markowitz model, which means that the variables and assumptions added to the index model are better than those of the Markowitz model.

5. Conclusion

This article intends to compare the portfolio analysis by implementing the Markowitz and Index models. Markowitz Model assumes that people are risk-averse, which means investors prefer a less risky portfolio than a riskier portfolio with the same expected return. The Index Model connects the capital asset pricing model, which predicts expected return and risk of an asset and realizes return. Both models are useful in real life, but there are still nuances in formulas, causing results to be different. This project determines the effect of two models' differences on results under five constraints. To get enough data for this portfolio, implementing a solver table is an efficient method to obtain the portfolio's return under each different standard deviation. After slightly modifying data that is obviously divergent, the final data is used to draw a graph.

After comparing results from two models, it is not hard to find that the Markowitz model's returns are always higher than the Index's ones under five constraints in the minimum variance graph. Also, in the maximum sharp ratio graph, the index model can always achieve a higher sharp ratio than Markowitz Model. Accordingly, choosing the Markowitz model could result in higher returns in a less risky portfolio, and choosing the Index model could cause a sharp ratio to be higher. Moreover, constraint three, which has no constraint, achieve the highest sharp ratio in both models because portfolio manager may choose some extremely risky stock like some small companies and choose to a short, large portion of the portfolio. These actions lead to a higher sharp ratio than all other constraints.

Although enormous data can prove results to be meaningful, there are still minor defects. Firstly, the two models are based on some assumptions that are impossible to be true in real life. For example, not all investors are risk-averse in real life, and people have different holding periods. Also, some points in the graph are not fully convergent.

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